Section 4.3 Implicit Differentiation (Minimum Homework: all odds)

#1-16: Use implicit differentiation to determine $\frac{dy}{dx}$.

1) $y^2 - 3x^2 = 4x - 3$

$$\frac{d}{dx}(y^2) - \frac{d}{dx}(3x^2) = \frac{d}{dx}(4x) - \frac{d}{dx}(3)$$

$$2y\frac{dy}{dx} - 6x = 4$$

$$+6x \quad +6x$$

$$\frac{2y}{2y}\frac{dy}{dx} = \frac{6x+4}{2y}$$

$$\frac{dy}{dx} = \frac{3x+2}{y} \text{ OR } \frac{3x}{y} + \frac{2}{y}$$

answer: $\frac{dy}{dx} = \frac{3x+2}{y}$

3) $5y - 2x^2 = 4x$

$$\frac{d}{dx}(5y) - \frac{d}{dx}(2x^2) = \frac{d}{dx}(4x)$$

$$5\frac{dy}{dx} - 4x = 4$$

$$+4x \quad +4x$$

$$\frac{5\frac{dy}{dx}}{5} = \frac{4x+4}{5}$$

$$\frac{dy}{dx} = \frac{4x+4}{5} \text{ OR } \frac{4x}{5} + \frac{4}{5}$$

answer: $\frac{dy}{dx} = \frac{4x+4}{5}$

5) $y^2 + 3y = 5x^2 + 3x + 1$

$$\frac{d}{dx}(y^2) + \frac{d}{dx}(3y) = \frac{d}{dx}(5x^2) + \frac{d}{dx}(3x) + \frac{d}{dx}(1)$$

$$2y\frac{dy}{dx} + 3\frac{dy}{dx} = 10x + 3$$

$$\frac{\frac{dy}{dx}(2y+3)}{2y+3} = \frac{10x+3}{2y+3}$$

$$\frac{dy}{dx} = \frac{10x+3}{2y+3}$$

Answer: $\frac{dy}{dx} = \frac{10x+3}{2y+3}$

7) $3y^2 - y = x^2 - 4x$

$$\frac{d}{dx}(3y^2) - \frac{d}{dx}(y) = \frac{d}{dx}(x^2) - \frac{d}{dx}(4x)$$

$$6y\frac{dy}{dx} - 1\frac{dy}{dx} = 2x - 4$$

$$\frac{dy}{dx}\frac{(6y-1)}{6y-1} = \frac{2x-4}{6y-1}$$

$$\frac{dy}{dx} = \frac{2x-4}{6y-1} \text{ OR } \frac{2(x-2)}{6y-1}$$

answer: $\frac{dy}{dx} = \frac{2x-4}{6y-1}$

9) $y^2 = 6y + x$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(6y) + \frac{d}{dx}(x)$$

$$2y\frac{dy}{dx} = 6\frac{dy}{dx} + 1$$

$$-6\frac{dy}{dx} \qquad -6\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - 6\frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\frac{(2y-6)}{2y-6} = \frac{1}{2y-6}$$

answer: $\frac{dy}{dx} = \frac{1}{2y-6} = \frac{1}{2(y-3)}$

$$\frac{dy}{dx} = \frac{1}{2y-6} \text{ OR } \frac{1}{2(y-3)}$$

11) $3y = y^2 + 4x - 3$

$$\frac{d}{dx}(3y) = \frac{d}{dx}(y^2) + \frac{d}{dx}(4x) - \frac{d}{dx}(3)$$

$$3\frac{dy}{dx} = 2y\frac{dy}{dx} + 4$$

$$-2y\frac{dy}{dx} \quad -2y\frac{dy}{dx}$$

$$-2y\frac{dy}{dx} + 3\frac{dy}{dx} = 4$$

$$\frac{dy}{dx}(-2y+3) = 4$$

$$\frac{-2y+3}{-2y+3} \qquad \frac{4}{-2y+3}$$

$$\frac{dy}{dx} = \frac{4}{-2y+3} \text{ OR } \frac{4}{-1(2y-3)} \text{ or } \frac{-4}{2y-3}$$

answer: $\frac{dy}{dx} = \frac{4}{-2y+3} = \frac{-4}{2y-3}$

13) $xy - 3x^2 = 5x$

$\frac{d}{dx}(xy) - \frac{d}{dx}(3x^2) = \frac{d}{dx}(5x)$

$x\frac{dy}{dx} + y - 6x = 5$

$-y \quad +6x \quad +6x - y$

$x\frac{dy}{dx} = 6x - y + 5$

$\frac{x}{x} \qquad \frac{x}{x}$

$\frac{dy}{dx} = \frac{6x - y + 5}{x}$

Or

$\frac{dy}{dx} = 6 - \frac{y}{x} + \frac{5}{x}$

$\frac{d}{dx}(xy)$

1ST x LAST y

DERIV 1 deriv 1 $\frac{dy}{dx}$

$x \cdot 1\frac{dy}{dx} + 1y$

$= x\frac{dy}{dx} + 1y$

answer $\frac{dy}{dx} = \frac{6x-y+5}{x}$

15) $5xy - 3x^2 = 5x^3$

$\frac{d}{dx}(5xy) - \frac{d}{dx}(3x^2) = \frac{d}{dx}(5x^3)$

$5x\frac{dy}{dx} + 5y - 6x = 15x^2$

$-5y + 6x \quad +6x - 5y$

$\frac{d}{dx}(5xy)$

1ST $5x$ 1ST y

Deriv 5 Deriv 1 $\frac{dy}{dx}$

$5x \cdot 1\frac{dy}{dx} + 5y$

$5x\frac{dy}{dx} + 5y$

$\frac{5x\frac{dy}{dx}}{5x} = \frac{15x^2 + 6x - 5y}{5x}$

$\frac{dy}{dx} = \frac{15x^2 + 6x - 5y}{5x}$

or $\frac{dy}{dx} = 3x + \frac{6}{5} - \frac{y}{x}$

answer: $\frac{dy}{dx} = \frac{15x^2+6x-5y}{5x}$

#17- 20: Find the equation of the line tangent to the graph at the indicated point. (Hint, these derivatives have been calculated above.)

17) $y^2 - 3x^2 = 4x - 3; (1,2)$ hint this was computed in problem 1: $\frac{dy}{dx} = \frac{3x+2}{y}$

$$m = \frac{3(1)+2}{2} = \frac{5}{2}$$

$$y - 2 = \frac{5}{2}(x-1)$$

$$y - 2 = \frac{5}{2}x - \frac{5}{2}$$

$$y - \frac{2}{1}(2)$$

$$y - \frac{4}{2} = \frac{5}{2}x - \frac{5}{2}$$

$$+\frac{4}{2} \qquad +\frac{4}{2}$$

$$y = \frac{5}{2}x - \frac{1}{2}$$

answer: $y = \frac{5}{2}x - \frac{1}{2}$

19) $xy - 3x^2 = 5x;\ \ (2, -1)$ hint this was computed in problem 13: $\frac{dy}{dx} = \frac{6x-y+5}{x}$

$$m = \frac{6(2)-(-1)+5}{2}$$

$$m = \frac{12+1+5}{2}$$

$$m = 18/2$$

$$m = 9$$

$$y-(-1) = 9(x-2)$$

$$y+1 = 9x-18$$

$$-1 \qquad -1$$

$$y = 9x-19$$

$answer$: $y = 9x - 19$

#21-28: Use implicit differentiation to determine $\frac{dr}{dt}$.

21) $C = 2\pi r$

$$\frac{d}{dt}(C) = \frac{d}{dt}(2\pi r)$$

$$\frac{1\,\frac{dC}{dt}}{2\pi} = \frac{2\pi\,\frac{dr}{dt}}{2\pi}$$

$$\frac{dC/dt}{2\pi} = \frac{dr}{dt}$$

answer: $\frac{dr}{dt} = \frac{\frac{dC}{dt}}{2\pi}$

23) $A = 5r^2$

$$\frac{d}{dt}(A) = \frac{d}{dt}(5r^2)$$

$$\frac{1\frac{dA}{dt}}{10r} = \frac{10r\frac{dr}{dt}}{10r}$$

$$\frac{dA/dt}{10r} = \frac{dr}{dt}$$

answer: $\frac{dr}{dt} = \frac{\frac{dA}{dt}}{10r}$

25) $V = 5 + 6r^2$

$$\frac{d}{dt}(V) = \frac{d}{dt}(5) + \frac{d}{dt}(6r^2)$$

$$\frac{dV}{dt} = 0 + 12r\frac{dr}{dt}$$

$$\frac{\frac{dV}{dt}}{12r} = \frac{12r\,dr/dt}{12r}$$

$$\frac{dV/dt}{12r} = \frac{dr}{dt}$$

answer: $\frac{dr}{dt} = \frac{\frac{dV}{dt}}{12r}$

27) $V = \frac{2}{3}\pi r^3$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{2}{3}\pi r^3\right)$$

$$\left(3 \cdot \frac{2}{3}\pi r^2 \quad 2\pi r^2\right)$$

$$\frac{dV}{dt}(1) = 2\pi r^2 \frac{dr}{dt}$$

$$\frac{dV/dt}{2\pi r^2} = \frac{2\pi r^2\, dr/dt}{2\pi r^2}$$

$$\frac{dV/dt}{2\pi r^2} = \frac{dr}{dt}$$

answer: $\frac{dr}{dt} = \frac{\frac{dV}{dt}}{2\pi r^2}$